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Quantum Gravity Effective Action Provides Entropy of The Universe

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References

- K. Hamada, Particles 7 (2024) 465, Special Issue Feature Papers for Particles 2023, arXiv:2401.09672
- K. Hamada, *Quantum Gravity and Cosmology Based on Conformal Field Theory* (Cambridge Scholar Publishing, Newcastle, 2018)

Introduction

What is The Origin of Cosmic Entropy?

The origin of a huge amount of cosmic entropy, most of which is currently carried by CMB, remains a mystery

- Within Einstein gravity, it cannot be satisfactorily explained without introducing an unknown scalar field as a source of all matter

- Standard cosmology assumes that entropy is conserved during evolution, which is usually explained as adiabatic process after big bang, but I wonder if it is really sufficient

- I think that the origin of entropy and its conservation were ruled by physical laws before big bang when universe was entangled in every corner

In this talk, I argue that entropy is derived from quantum spacetime states described by renormalizable quantum gravity with background freedom in UV limit

General Properties of Renormalizable Theory

In general, effective action in renormalizable quantum field theory is finite and is renormalization group (RG) invariant

Energy-momentum tensor is also finite (normal product) and RG invariant

If we consider ordinary quantum field theory with diffeomorphism invariance, then

$$\frac{\delta}{\delta g_{\mu\nu}(x)} \int [df]\, e^{iI_{\rm M}} = i\frac{1}{2}\sqrt{-g}\,\langle T^{\mu\nu}_{\rm M}(x)\rangle = \text{finite}$$

f is an ordinary matter field in curved spacetime

Furthermore, when gravity is quantized, total energy-momentum tensor vanishes, that is, Hamiltonian and momentum constraints

$$\int [dg] \frac{\delta}{\delta g_{\mu\nu}(x)} \left(\int [df]\, e^{iI} \right) = i\frac{1}{2}\langle \sqrt{-g}\, T^{\mu\nu}(x)\rangle = 0 \qquad I = I_{\rm G} + I_{\rm M}$$

Schwinger-Dyson equation (= quantum-mechanical identity)

Strictly speaking, we need to define path integral measure (shown later), but captures the essence why Hamiltonian vanishes

Entropy of The Universe

Partition function of quantum gravity represents sum over all states of quantum spacetime, and effective action is given by its logarithm

Since total Hamiltonian vanishes, quantum gravity effective action provides statistical entropy for states of universe:

$$\Gamma_{\mathrm{QG}} = -S_{\mathrm{Univ}}$$

conserved as an RG invariant, that is, physical constant

This formula is a general conclusion derived from diffeomorphism invariance and renormalizability

It may be helpful to recall that effective action corresponds to free energy in ordinary quantum field theory

If we impose a condition for energy to disappear for that, we can formally obtain above relation

Contents

How To Define Renormalizable Theory of Gravity

Problems With Quantizing Einstein Gravity

- Coupling constant (Newton constant) has dimension

  ➔ perturbation theory is not renormalizable

- Einstein-Hilbert (EH) action is not bounded below

  ➔ the theory is unstable, not well-defined, even in non-perturbative way

- There exist spacetime singularities

  Schwarzschild solution is Ricci flat

  ➔ EH action vanishes for singularity

  ➔ Path integral weight becomes unity, or finite, and thus the singularity cannot be excluded statistically, namely physical, like soliton and instanton

This is linked with the fact that Einstein eq. $M_{\rm P}^2(-R_{\mu\nu}+g_{\mu\nu}R/2)+T^{\rm M}_{\mu\nu}=0$ does not contain Riemann tensor corresponding to field strength of gravity, so has no sufficient ability to control curvature, unlike gauge field eq. $-\nabla^\mu F_{\mu\nu}+J_\nu=0$

Therefore, Einstein gravity cannot exceed Planck scale

Planck Scale Wall and Renormalizablility

Planck scale wall:

Einstein gravity has singularities and is not renormalizable

Usually, in order to avoid such problems, UV cutoff is introduced in Planck scale

Some people think of it as an entity of spacetime quantization

But, introducing finite UV cutoff breaks diffeomorphism inv.

Many people believe that there is no world shorter than Planck length, or that such a world is ruled by a physical law other than diffeomorphism inv.

However, this thinking is exactly root cause of problems with gravity

singularity, renormalizability, unitarity, cosmological constant problem, entropy of universe

Bringing UV cutoff to infinity, or continuum limit, is to make quantum theory of gravity not only diffeomorphism invariant but also renormalizable

Early Attempts and Setback

In 1970s, in order to overcome problems with EH action, fourth-derivative actions, $R^2_{\mu\nu\lambda\sigma}$, $R^2_{\mu\nu}$, R^2 , are introduced

In general, fourth-derivative quantum gravity has the following good properties:

- Renormalizable (coupling constant is dimensionless)
- Action is positive-definite (bounded below) ➔ path integral is stable (unlike EH)
- Singularities are forbidden as unphysical objects

If action contains Riemann tensor squared, then it diverges for singularities

Note) Path integral weight for singularities (Wick rotated) :

Einstein gravity : $e^{-\int R} = 1$, while 4th-deriv. gravity : $e^{-\int R^2_{\mu\nu\lambda\sigma}} = 0$ ← statistically forbidden!

In this way, many drawbacks in Einstein gravity can be overcome by introducing fourth-derivative gravitational actions

Despite these advantages, however, if it is quantized using weak-field approxiamtion (= graviton picture) as usual, ghost modes arise as physical particles

Beyond Weak-Field Approximation

Magnitude of observed graviton is

$$\delta L/L \sim 10^{-20}$$

while in trans-Planckian world

$$\delta L/L \sim o(1)$$

Ghost problem is caused by weak-field (graviton) approximation:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + H_{\mu\nu} \qquad\qquad H_{\mu\nu} \ll 1$$

Entire spacetime structure $\hat{g}_{\mu\nu}$ is determined by $\hat{T}^{\mu\nu} = 0$ (usually $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$), whereas quantum field $H_{\mu\nu}$ is treated as Hamiltonian eigenstate with non-zero eigenvalue on $\hat{g}_{\mu\nu}$

Weak-field approximation is nothing more than reducing gravitational system to a system of special relativity, or Lorentz invariant system

It is an ordinary quantum-mechanical system, in which case vanishing of Hamiltonian is not considered

This approximation can only apply to local particle worlds, where back-reactions are too weak to affect entire spacetime structure and can therefore be neglected

Not adequate for trans-Planckian world where spacetime itself fluctuates greatly

Therefore, quantum gravity should be defined non-perturbatively by quantizing the field itself to obey Hamiltonian and momentum constraints

Planck scale physics begins by discarding particle picture, or Lorentz inv. picture!

Renormalizable and Asymptotically Background-Free Quantum Gravity

Novel Perturbation Method

Inflation gives a hint of how to formulate quantum theory of gravity

Inflationary spacetime = conformally flat (de Sitter) spacetime

⇩

Quantum spacetime should be described by perturbation expansion around conformally-flat spacetime where Weyl tensor $C_{\mu\nu\lambda\sigma}$ vanishes:

$$g_{\mu\nu} = \underline{e^{2\phi}}\bar{g}_{\mu\nu} \qquad \bar{g}_{\mu\nu} = \eta_{\mu\lambda}\left(e^h\right)^\lambda{}_\nu = \eta_{\mu\lambda}\left(\delta^\lambda{}_\nu + h^\lambda{}_\nu + \frac{1}{2}h^\lambda{}_\sigma h^\sigma{}_\nu + \cdots\right)$$

This part is not restricted from $C_{\mu\nu\lambda\sigma} = 0$ condition

Traceless tensor mode *h* is handled in perturbation by introducing coupling constant later

Most important conformal factor determining distance is treated non-perturbatively

➔ realize background freedom as a special conformal invariance in UV limit

(unlike conventional perturbation method defined around flat spacetime of $R_{\mu\nu\lambda\sigma} = 0$)

Renormalizable Quantum Gravity Action

Quantum gravity action that becomes conformally invariant
in a trans-Planckian region :

$$I = \int d^4x \sqrt{-g} \left[\underbrace{-\frac{1}{t^2} C_{\mu\nu\lambda\sigma}^2 - bG_4}_{\text{Conformally invariant (no } R^2)} + \frac{1}{\hbar} \left(\frac{1}{16\pi G} R - \Lambda + \mathcal{L}_{\mathrm{M}} \right) \right]$$

conformal in UV

where $C_{\mu\nu\lambda\sigma}$ is Weyl tensor and G_4 is Euler density, and $\mathrm{sgn} = (-1, 1, 1, 1)$

II

field strength of $h^\lambda{}_\nu$

Coupling constant *t* controls expansion around $C_{\mu\nu\lambda\sigma} = 0$, which is dimensionless and renormalizable

[*b* is not independent coupling const. because G_4 has no kinetic term]

(c.f. $-\frac{1}{\mathfrak{g}^2} \mathrm{Tr} F_{\mu\nu}^2$ ⇨ expansion around $F_{\mu\nu} = 0$)

Note) $\hbar$ appears only before lower actions, because 4th-derivative gravity actions are exactly dimensionless, and so they contribute only to quantum dynamics, have no classical entity

➔ may be regarded as part of path integral measure

Diffeomorphism Invariant Measure

Unlike weak-field approximation, path integral measure has conformal-factor dependence:

$$\int [dg]_g \, e^{iI(g)} = \int [d\phi dh]_{\underline{\underline{\eta}}} \, e^{\underline{iS(\phi,\bar{g})}+iI(g)}$$

diff. inv. measure

practical measures defined on flat metric so that normal field theory techniques can be applied

The *S* (=Jacobian) arises to ensure diffeomorphism invariance, which is Wess-Zumino action for conformal anomaly

[# physical quantity against the name]

Even at UV (t = 0) limit, *S* exists, that is Riegert action (= kinetic term of ϕ)

$$S_{\rm R} = \int d^4x \left\{ -\frac{b_1}{(4\pi)^2} \left[2\phi\bar{\Delta}_4\phi + \left(\bar{G}_4 - \frac{2}{3}\bar{\nabla}^2\bar{R} \right) \phi \right] \right\}$$

4th order conf. inv. differential. op

(c.f. Liouville action in 2D QG)

Interaction terms like $\phi^{n+1}\bar{\Delta}_4\phi$, $\phi^n\bar{C}^2_{\mu\nu\lambda\sigma}$, $\phi^n\bar{F}^2_{\mu\nu}$ arise at higher order of *t*

Both Singularities and Ghosts are Unphysical

This quantum gravity has the following good properties:

- Renormalizable (coupling constant *t* is dimensionless)
- Both Weyl and Riegert actions are bounded below, namely stable
- Singularities can be eliminated as unphysical because actions diverge for them

Furthermore, it has the following extra good property:

BRST conformal inv. arises as a manifestation of diffeomorphism inv. in UV limit

This represents background freedom where all of different conformally-flat spacetimes are gauge equivalent, unlike normal conformal inv.

BRST condition (= Hamiltonian and momentum constraints)

$Q_{\rm B}|{\rm phys}\rangle = 0$ Physical states exist in an infinite number ➔ can produce entropy

- All of ghost modes in fields become unphysical, namely, not BRST invariant
- Physical states are given by real composite primary scalars, while tensor states are forbidden, consistent with CMB observations (no tensor mode initially)

Diffeomorphism Inv., Entropy, and Ghosts

In renormalizable QG, energy-momentum tensor vanishes as an identity:

$$\int [dg] \frac{\delta}{\delta g_{\mu\nu}(x)} e^{iI} = i\frac{1}{2}\langle \sqrt{-g}\, T^{\mu\nu}(x) \rangle = 0$$

Schwinger-Dyson equation

Ghost modes are then necessary for Hamiltonian to vanish

Note that Einstein gravity also has a ghost mode causing unboundedness of EH action, but it allows non-trivial solutions such as Friedmann solution, gravitational objects, formation of large-scale structure, and so on

If all modes consist of physical ones, the only state in which total Hamiltonian vanishes is trivial vacuum ➔ no entropy and no time

Existence of ghost modes itself is not the problem!
Ghost modes cause problems only when they arise as physical objects

In this QG, ghost modes exist, despite that action is positive-definite, but they all become unphysical under BRST conformal inv.

Hence, ghost modes are essential in classical and quantum gravitational systems, but never be seen as physical

Effective Action, Equations of Motion, and Inflationary Solution

Effective Action for Weyl Sector

Effective action can be written in terms of running coupling constant

$$\Gamma^{\mathrm{W}} = -\left[\frac{1}{t^2} - 2\beta_0\phi + \beta_0 \log\left(\frac{q^2}{\mu^2}\right)\right]\sqrt{-g}C^2_{\mu\nu\lambda\sigma}$$

$$= -\frac{1}{\bar{t}^2(Q)}\sqrt{-g}C^2_{\mu\nu\lambda\sigma}$$

q = momentum measured by flat metric
like comoving mom. in cosmology

WZ action of conf. anomaly for Weyl sector,
necessary to preserve diff. inv.

$$\beta_0 = \{(N_X + 3N_W + 12N_A)/240 + 197/60\}/(4\pi)^2$$

Running coupling constant

$$\bar{t}^2(Q) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{\mathrm{QG}})}$$

where $Q^2 = g^{\mu\nu}q_\mu q_\nu = \dfrac{q^2}{e^{2\phi}}$

physical momentum squared

New physical scale (= RG inv. $d\Lambda_{\mathrm{QG}}/d\mu = 0$)

$$\Lambda_{\mathrm{QG}} \left(= \mu e^{-1/2\beta_0 t^2}\right)$$

determined from QG inflation scenario later

In general, nonlocal corrections are incorporated into the form of replacing t^2 with $\bar{t}^2(Q)$ in this way

This form holds even at higher loops, although $\bar{t}^2(Q)$ becomes more complicated

Effective Action for Riegert Sector

The coefficient of Riegert action has quantum corrections as $b_1 = b_c B$, so that

$$\Gamma^{\mathrm{R}} = \int d^4x \left\{ -\frac{b_c}{(4\pi)^2} B \left[2\phi \bar{\Delta}_4 \phi + \left(\bar{G}_4 - \frac{2}{3} \bar{\nabla}^2 \bar{R} \right) \phi \right] \right\}$$

(←only ϕ-dep. part is present here)

where $b_c = (N_X + 11N_W/2 + 62N_A)/360 + 769/180$

For Standard Model, $b_c = 7.0$

(For SU(5) GUT, $b_c = 9.1$)

Correction factor *B* is assumed to be summed up in the following:

$$B = 1 - \gamma_1 \frac{t^2}{4\pi} + \cdots \to \bar{B}(Q) = \left[1 + \gamma_1 \frac{\bar{t}^2(Q)}{4\pi} \right]^{-1}$$

[nonlocal corrections are incorporated by replacement $t^2 \to \bar{t}^2(Q)$]

This factor, called dynamical factor, expresses that conformal gravity dynamics disappear at dynamical scale

That is, at $Q = \Lambda_{\mathrm{QG}}$, running coupling diverges, then both Γ^{W} and Γ^{R} vanish

cf. gluon dynamics disappears at QCD scale as

$$\Gamma^{\mathrm{A}} = -\frac{1}{\bar{g}^2(Q)} \mathrm{Tr}(F_{\mu\nu}^2) \to 0 \quad \text{when} \quad \bar{g}^2(Q) \to \infty$$

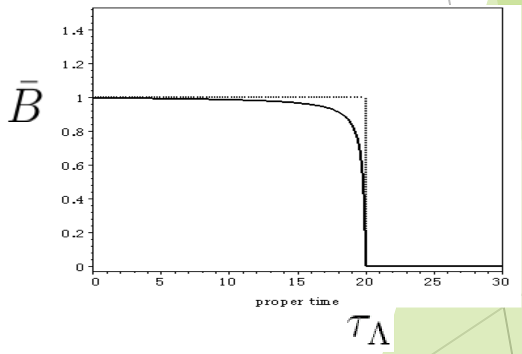
Dynamical Model of Spacetime Phase Transition

In order to describe time evolution of the universe, consider homogeneous component $\hat{\phi}$, average of spacetime fluctuations

Introduce physical (proper) time $d\tau = a d\eta$, where $a = e^{\hat{\phi}}$ is scale factor

Running coupling constant is approximated in time-dependent mean field by making replacement $Q^2 \to 1/\tau^2$ as

$$\bar{t}^2(\tau) = [\beta_0 \log(\tau_\Lambda^2/\tau^2)]^{-1}$$



Running coupling diverges at dynamical time $\tau = 1/\Lambda_{\rm QG}\ (\equiv \tau_\Lambda)$

Thus, inverse of running coupling and dynamical factor vanishes there

In this way, <u>disappearance</u> of conformal gravity dynamics, $\Gamma^{\rm W}$ and $\Gamma^{\rm R}$, at phase transition point is described

$$\delta\Gamma = \int d^4x \left(\mathbf{T}^{\mu}{}_{\mu}\delta\phi + \frac{1}{2}\mathbf{T}^{\mu}{}_{\nu}\delta h^{\nu}{}_{\mu} \right) = 0$$

Stable Inflationary Solution

Inflationary solution exists when $H_{\mathrm{D}} > \Lambda_{\mathrm{QG}}$, and here $N = \dfrac{H_{\mathrm{D}}}{\Lambda_{\mathrm{QG}}} = 60$, $\quad H_{\mathrm{D}} = \sqrt{\dfrac{8\pi^2}{b_c}} M_{\mathrm{P}}$

$M_{\mathrm{P}}\,(=1/\sqrt{8\pi G}) < H_{\mathrm{D}} < m_{\mathrm{pl}}\,(=1/\sqrt{G})$ $\quad b_c = 7.0$

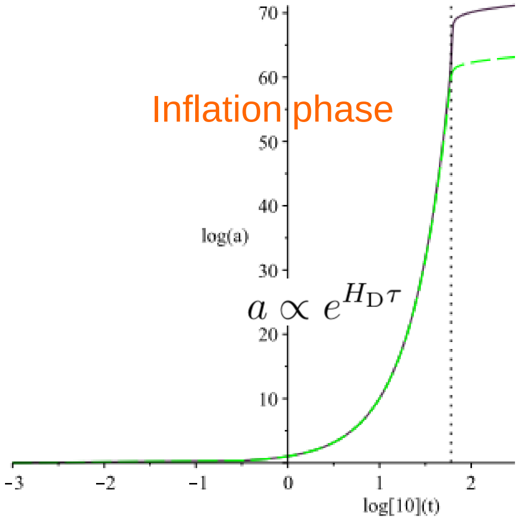
Homogeneous equation of motion ($\mathbf{T}^{\mu}{}_{\mu} = 0$)

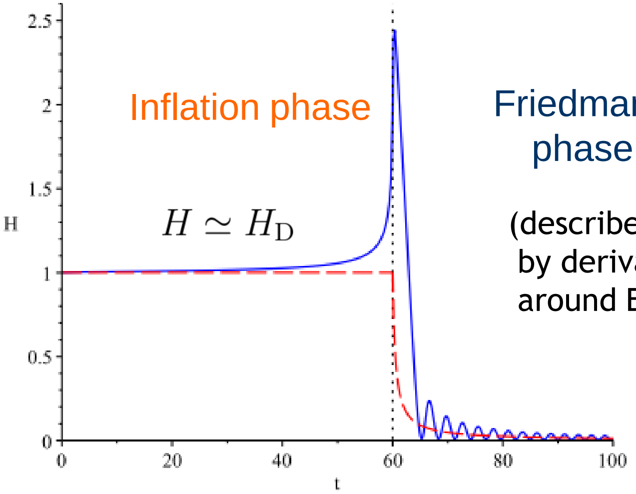
$$-\frac{b_c}{4\pi^2}\bar{B}(\tau)\partial_\eta^4\hat{\phi} + 6M_{\mathrm{P}}^2\, e^{2\hat{\phi}}\left(\partial_\eta^2\hat{\phi} + \partial_\eta\hat{\phi}\partial_\eta\hat{\phi}\right) = 0$$

$$\Rightarrow \quad \bar{B}(\tau)\left(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 18H^2\dot{H} + 6H^4\right) - 3H_{\mathrm{D}}^2\left(\dot{H} + 2H^2\right) = 0$$

Hubble variable: $H = \partial_\tau a/a$

Inflation starts at Planck time ($\tau_{\mathrm{P}} = 1/H_{\mathrm{D}}$) and ends at dynamical time ($\tau_\Lambda = 1/\Lambda_{\mathrm{QG}}$)





(described by LEFT defined by derivative expansion around Einstein gravity)

H_{D} is normalized to unity

Dashed lines are $H = H_{\mathrm{D}}$ and Friedmann solutions

Energy Conservation and Big Bang

Energy conservation equation ($\mathbf{T}_{00} = 0$)

matter energy density

$$\frac{b_c}{8\pi^2}\bar{B}(\tau)\left(2\partial_\eta^3\hat{\phi}\partial_\eta\hat{\phi} - \partial_\eta^2\hat{\phi}\partial_\eta^2\hat{\phi}\right) - 3M_\mathrm{P}^2 e^{2\phi}\partial_\eta\hat{\phi}\partial_\eta\hat{\phi} + e^{4\hat{\phi}}\rho = 0$$

$$\Rightarrow \quad \bar{B}(\tau)\left(2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H} + 3H^4\right) - 3H_\mathrm{D}^2 H^2 + \frac{8\pi^2}{b_c}\rho = 0$$

energy shift as *B* decreases

Inflationary solution $H \simeq H_\mathrm{D}$ indicates $\rho \simeq 0$, thus no matter initially

At transition point $\tau_\Lambda = 1/\Lambda_\mathrm{QG}$, dynamical factor *B* vanishes and then gravitational energy shifts to matter density ρ, causing big bang

Interactions that create matter density is given by Wess-Zumino actions such as $\phi F_{\mu\nu}^2$ and $\phi\, C_{\mu\nu\lambda\sigma}^2$, which are open as $\bar{t}^2(\tau)$ increases near phase transition

Gravitons are also generated at low energy because BRST conf. inv. is broken there
At that time, due to $M_\mathrm{P} > \Lambda_\mathrm{QG}$, massive gravitons are never generated

Determination of Dynamical Scale

Number of e-foldings is

$$\mathcal{N}_e = \log \frac{a(\tau_\Lambda)}{a(\tau_{\rm P})} \simeq \frac{H_{\rm D}}{\Lambda_{\rm QG}} \quad \sim 60 - 70$$

Amplitude at transition point is roughly estimated as

$$\left.\frac{\delta R}{\hat{R}}\right|_{\tau_\Lambda} \sim \frac{\Lambda_{\rm QG}^2}{12 H_{\rm D}^2} \quad \sim 10^{-4} - 10^{-5}$$ from CMB observation

de Sitter curvature

$$\Lambda_{\rm QG} \simeq 10^{17}\ {\rm GeV}$$

(Amplitude reduction, or stability, is from positivity of 4-derivative action)

Comoving dynamical scale

$$\lambda = \Lambda_{\rm QG}/a_0$$ where a_0 is current scale factor, while initial a is set to be 1

This scale can explain sharp falloff at low multipoles of CMB angular power spectrum:

$$\lambda \sim H_0$$

Hubble constant

If setting $\lambda = 0.00023\,{\rm Mpc}^{-1}$ $\Rightarrow$ $a_0 = 10^{59}$ current scale factor

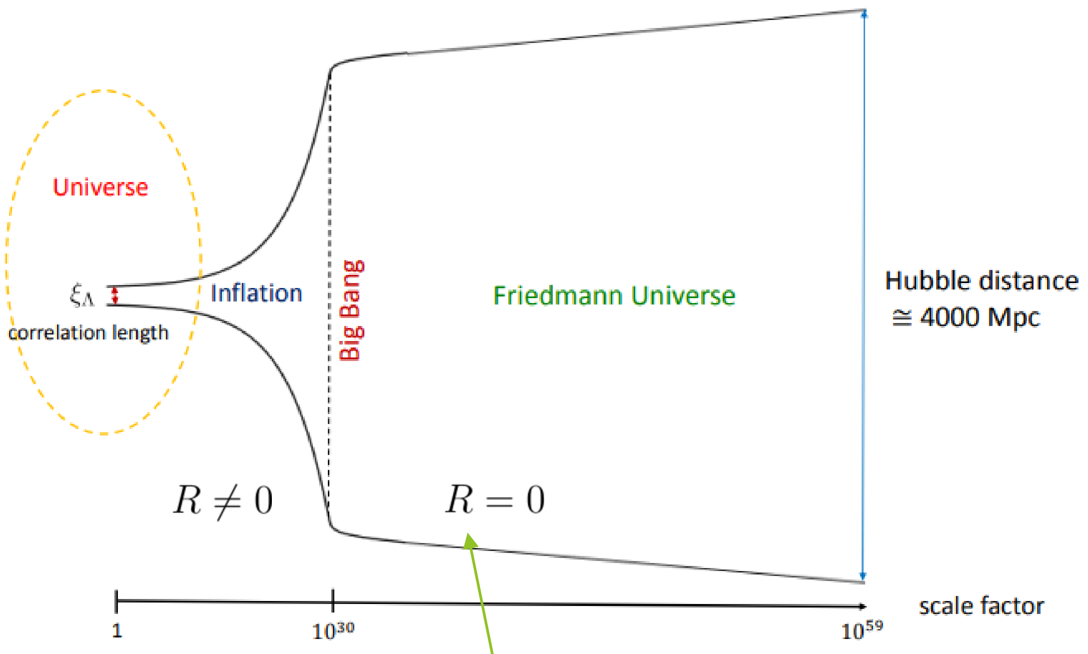
(Note) BRST conf. inv. suggests there are no primordial tensor fluctuations involved in CMB
In quantum gravity inflation, tensor-to-scalar ratio does not give limit on inflation scale)

Quantum Gravity Inflation Scenario

Correlation length of quantum gravity has expanded to Hubble distance

$1/H_0 \simeq 10^{59}\xi_\Lambda$ or $H_0 \simeq \lambda$ can explain sharp falloff at low multipole of CMB spectrum

$\xi_\Lambda = 1/\Lambda_{\mathrm{QG}}$



scalar curvature is order parameter

Entropy of The Universe

Entropy of Quantum Spacetime

Most of entropy in quantum spacetime is carried by conformal mode, rather than tensor or matter modes

Homogeneous component of conformal mode is an average value of spacetime fluctuations, and is responsible for entropy significantly

Effective action for this mode is

$$\begin{aligned}\Gamma_{\mathrm{QG}} &= V_3 \int d\eta \Big\{ -\frac{b_c}{8\pi^2} B\hat{\phi}\partial_\eta^4\hat{\phi} + 3M_{\mathrm{P}}^2 e^{2\hat{\phi}} (\partial_\eta^2\hat{\phi} + \partial_\eta\hat{\phi}\partial_\eta\hat{\phi}) \Big\} \\ &= -3M_{\mathrm{P}}^2 V_3 \int d\eta (\hat{\phi} - 1) e^{2\hat{\phi}} (\partial_\eta^2\hat{\phi} + \partial_\eta\hat{\phi}\partial_\eta\hat{\phi})\end{aligned}$$

where $V_3 = \int d^3\mathbf{x}$ initial spatial volume

Since $C_{\mu\nu\lambda\sigma} \simeq 0$ during most of inflation period, contributions from Weyl part to entropy is sufficiently small and can be ignored

Main role of Weyl sector is to transfer entropy carried by conformal mode to matter fields at spacetime phase transition

Estimation of Entropy

In order to carry out calculations analytically, here consider a radically simplified model in which $\bar{t}^2$ remains almost zero and diverges sharply at τ_Λ , so that $\bar{B}$ is almost 1 and abruptly vanishes at τ_Λ as a step function

From $S_{\rm Univ} = -\Gamma_{\rm QG}$, entropy is evaluated as

$$
\begin{aligned}
S_{\rm Univ} &= 6M_{\rm P}^2 H_{\mathbf{D}}^2 V_3 \int_0^{\tau_\Lambda} d\tau e^{3H_{\mathbf{D}}\tau}(H_{\mathbf{D}}\tau - 1) \\
&= 2M_{\rm P}^2 H_{\mathbf{D}} V_3 \left[e^{3H_{\mathbf{D}}\tau_\Lambda}\left(H_{\mathbf{D}}\tau_\Lambda - \frac{4}{3}\right) + \cancel{\frac{4}{3}} \right]
\end{aligned}
$$

negligible

$$
H_{\rm D} = \sqrt{\frac{8\pi^2}{b_c}} M_{\rm P}
$$

Rewriting in terms of $N = \dfrac{H_{\rm D}}{\Lambda_{\rm QG}}$, where $\mathcal{N}_e = N$ in radically simplified model

$$
S_{\rm Univ} = 2\sqrt{\frac{8\pi^2}{b_c}} M_{\rm P}^3 e^{3N}\left(N - \frac{4}{3}\right) V_3
$$

Comparison with Current Entropy

Current entropy density (from Kolb-Turner book) is

$$s = \frac{S_{\rm Univ}}{V_3^{\rm today}} = \frac{2\pi^2}{45} g_{*S} T^3 \simeq 2.91 \times 10^3 \,\mathrm{cm}^{-3}$$

$$g_{*S} = 3.91$$
$$T = 2.73\,\mathrm{K} = 11.9\mathrm{cm}^{-1}$$

The universe expands by a_0 from before inflation to present

$$V_3^{\rm today} = a_0^3 V_3$$

Therefore, corresponding QG entropy density is

$$s = 2\sqrt{\frac{8\pi^2}{b_c}} M_{\rm P}^3 e^{3N} \left(N - \frac{4}{3}\right) a_0^{-3}$$

Fixing the parameters to $b_c = 7$ and $a_0 = 10^{59}$ and finding the value of N so that this agrees with current entropy density, $N = 62.2$ is obtained

Current cosmic entropy is consistent with scenario of quantum gravity inflation

More Realistic Case

Ratio between $2.73\mathrm{K}$ and $\Lambda_{\mathrm{QG}} \simeq 10^{17}\,\mathrm{GeV}$ suggests that the universe expands about 10^{29} times after settling into Friedmann spacetime

Therefore, to make totally 10^{59}, the universe has to expand about 10^{30} times during inflation era

This corresponds to $\mathcal{N}_e \simeq 70$, which is greater than 60

This contradiction can be resolved by considering a more realistic dynamical model described by time-dependent running coupling constant

In fact, current entropy can be explained by setting dynamical parameters as

$$\beta_0 = 0.171 \text{ and } \gamma_1 = 0.1, \text{ with } N = 60$$

These values are not so important because result is dynamical-model dependent

What is important here is that quantum gravity can generate sufficient entropy!

Conclusion

Quantum Gravity Origin of Cosmic Entropy

Diffeomorphism invariance and renormalizability concludes that quantum gravity effective action gives entropy because Hamiltonian vanishes such that

$$\Gamma_{\mathrm{QG}} = -S_{\mathrm{Univ}}$$

and is conserved as RG invariant

Renormalizable quantum gravity with asymptotic background freedom (ABF) can produce sufficient entropy of the universe

At that time, ghost modes are necessary for entropy to exist, because $H = 0$ solution would be trivial vacuum if all modes were physical

Diffeomorphism Invariance and Unitarity

Existence of ghost modes itself is not problem

In Einstein gravity, ghost mode that prevents boundedness of EH action causes instability to form large-scale structure of the universe
(although many people do not pay attention to this fact)

In ABF quantum gravity, ghost modes arise as well, but situation is very different

Fourth-derivative action is positive-definite and therefore stable, so that spacetime fluctuations reduce in amplitude, leading to proper initial conditions of Friedmann universe [K.H., arXiv:2306.01384]

If gravitational field is handled as it is without separating positive- and negative-metric modes, then positivity of field action ensures reality of the field, or unitarity in terms of CFT

Recall no particle states, thus no *S*-matrix, in trans-Planckian region

Hence, if there is a constraint that does not allow separation of field into such modes, reality of field is preserved ➔ BRST conformal invariance exactly ensures this

Make all ghost modes unphysical, not gauge inv.

Diffeomorphism Invariance and Ghosts

Ghost modes must be invisible, but necessary in classical and quantum gravitational systems

In the end, root cause of unitarity issue comes from application of weak-field (graviton) approximation to trans-Planckian physics

That is nothing more than reducing gravitational system to a system of special relativity, or Lorentz invariant system,

which is an ordinary quantum mechanical system where Hamiltonian eigenstates with non-zero eigenvalues are considered, in which back-reactions are neglected

When constructing quantum gravity, we cannot consider diffeomorphism invariance as an extension of Lorentz invariance

Lorentz inv. forbids ghosts, whereas diffeomorphism inv. requires ghosts

Hamiltonian eigenstates (approximation formula for gravity)

H = 0 state (identity)

$$\int [dg] \frac{\delta}{\delta g_{\mu\nu}(x)} e^{iI} = i\frac{1}{2}\langle \sqrt{-g}\, T^{\mu\nu}(x) \rangle = 0$$

See Books for Mathematical Details on Renormalization and BRST conformal inv.

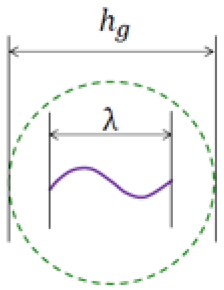
共形場理論を基礎にもつ量子重力理論と宇宙論
(プレアデス出版、2016)

Quantum Gravity and Cosmology Based on Conformal Field Theory
(Cambridge Scholars Publishing, 2018)

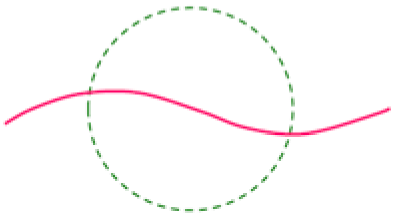
Appendix

Collapse of Point-Particle Picture

Particle with mass exceeding Planck mass is nothing but a black hole



$m > m_{pl}$



$m < m_{pl}$

Compton wavelength (= particle size)

$$\lambda \sim 1/m$$

Horizon size

$$h_g \sim m/m_{\rm pl}^2$$

Particle information is hidden inside horizon and lost beyond Planck scale

⇨ Unitarity is broken at Planck scale

Particle picture is not justified there

Diffeomorphism Invariance and Entropy

Effective action of renormalizable quantum gravity with asymptotic background freedom can provide entropy of the universe

Note here that there are no ordinary particle states in quantum gravity, thus there is no concept of temperature or thermal equilibrium

Thermal states can define after matter is created at spacetime phase transition

Formula $\Gamma_{\rm QG} = -S_{\rm Univ}$ represents purely counting number of quantum gravity states, where no temperature and vanishing Hamiltonian are complementary

Simplicial quantum gravity, lattice version of this theory, is defined by counting possible spacetime configurations

Background Freedom as BRST Conformal Inv.

Background freedom arises in UV limit of $t \to 0$ as part of diffeomorphism invariance $\delta_\xi g_{\mu\nu} = g_{\mu\lambda}\nabla_\nu \xi^\lambda + g_{\nu\lambda}\nabla_\mu \xi^\lambda$, in which ξ^λ is given by conformal Killing vectors c^λ :

[other gauge d.o.f. are fixed, e.g. in radiation gauge]

$$\delta_{\rm B}\phi = c^\mu \partial_\mu \phi + \frac{1}{4}\partial_\mu c^\mu$$

$$\delta_{\rm B} h_{\mu\nu} = c^\lambda \partial_\lambda h_{\mu\nu} + \frac{1}{2} h_{\mu\lambda}\left(\partial_\nu c^\lambda - \partial^\lambda c_\nu\right) + \frac{1}{2} h_{\nu\lambda}\left(\partial_\mu c^\lambda - \partial^\lambda c_\mu\right)$$

This conformal symmetry is a gauge symmetry, not a normal one

All theories with different backgrounds connected to each other by conformal transformations are gauge equivalence

||

Independence of how to choose background metric

$$\hat{g}_{\mu\nu} \cong e^{2\omega(x)}\hat{g}_{\mu\nu}$$

[That for tensor mode is less dominant]

In other words, since $g_{\mu\nu} = e^{2\phi}\bar{g}_{\mu\nu}$, a conformal change of $\hat{g}_{\mu\nu}$ can be absorbed by a shift change of ϕ , while ϕ is an integration variable and its measure is invariant under the shift so that the theory does not change ← performing integration is essential

Wess-Zumino Consistency Condition and Background Freedom

Integral representation of Riegert action

$$S_{\mathrm{R}}(\phi,\bar{g}) = -\frac{b_1}{(4\pi)^2}\int d^4x \int_0^\phi d\phi \sqrt{-g}\, E_4 = -\frac{b_1}{(4\pi)^2}\int d^4x \sqrt{-\bar{g}}\,(2\phi\bar{\Delta}_4\phi + \bar{E}_4\phi)$$

Wess-Zumino consistency condition

$$E_4 = G_4 - 2\nabla^2 R/3$$

$$S_{\mathrm{R}}(\phi-\omega, e^{2\omega}\bar{g}) + S_{\mathrm{R}}(\omega,\bar{g}) = S_{\mathrm{R}}(\phi,\bar{g})$$

Proof of background-metric independence

$$\begin{aligned} Z(e^{2\omega}\hat{g}) &= \int [d\phi dh]_{e^{2\omega}\hat{g}}\, e^{iS_{\mathrm{R}}(\phi,e^{2\omega}\bar{g})+iI(e^{2\omega}g)} \\ &= \int [d\phi dh]_{\hat{g}}\, e^{iS_{\mathrm{R}}(\omega,\bar{g})} e^{iS_{\mathrm{R}}(\phi,e^{2\omega}\bar{g})+iI(e^{2\omega}g)} \\ &= \int [d\phi dh]_{\hat{g}}\, e^{\underbrace{iS_{\mathrm{R}}(\omega,\bar{g})+iS_{\mathrm{R}}(\phi-\omega,e^{2\omega}\bar{g})}+iI(g)} \\ &= Z(\hat{g}) \end{aligned}$$

use Wess-Zumino consistency condition

Low Energy Effective Gravity Theory

Dynamics of 4th-derivative conformal gravity disappears below Λ_{QG}, and there ϕ and $h^{\mu}{}_{\nu}$ are tightly binding

Dynamical variable at low energy is given by composite metric tensor composed of these modes, which is ruled by lower-derivative action, namely Einstein action

Low EFT is given by derivative expansion about Einstein theory

$$I_{\mathrm{low}} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_2 + \mathcal{L}_4 + \cdots \right\} \qquad \mathcal{L}_2 = \frac{M_{\mathrm{P}}^2}{2} R + \mathcal{L}_2^{\mathrm{M}}$$

Using lowest Einstein equation $M_{\mathrm{P}}^2 R_{\mu\nu} = T_{\mu\nu}^{\mathrm{M}}$, 4th-derivative terms can be reduced to

$$\mathcal{L}_4 = \frac{\alpha}{(4\pi)^2} R^{\mu\nu} R_{\mu\nu}$$

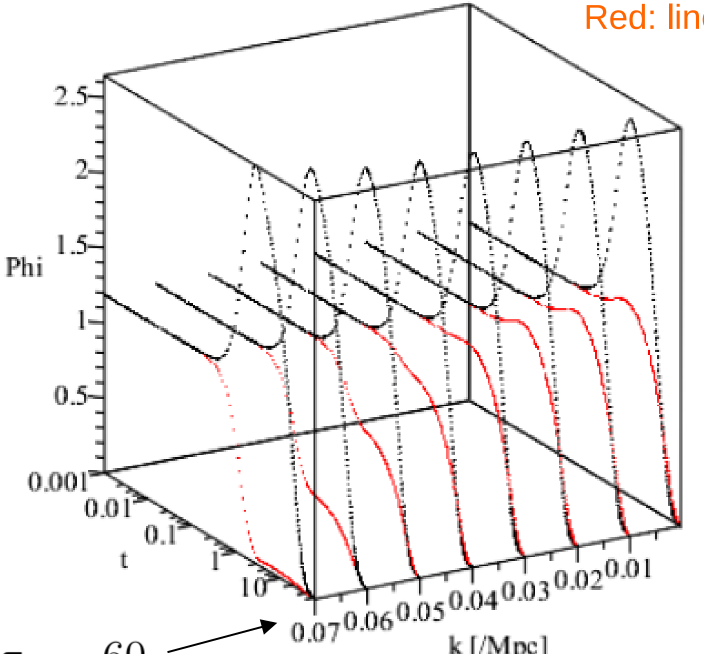
phenomenological parameter

Reduction of Spacetime Fluctuations

[K.H., Universe 10 (2024) 33, arXiv:2306.01384]

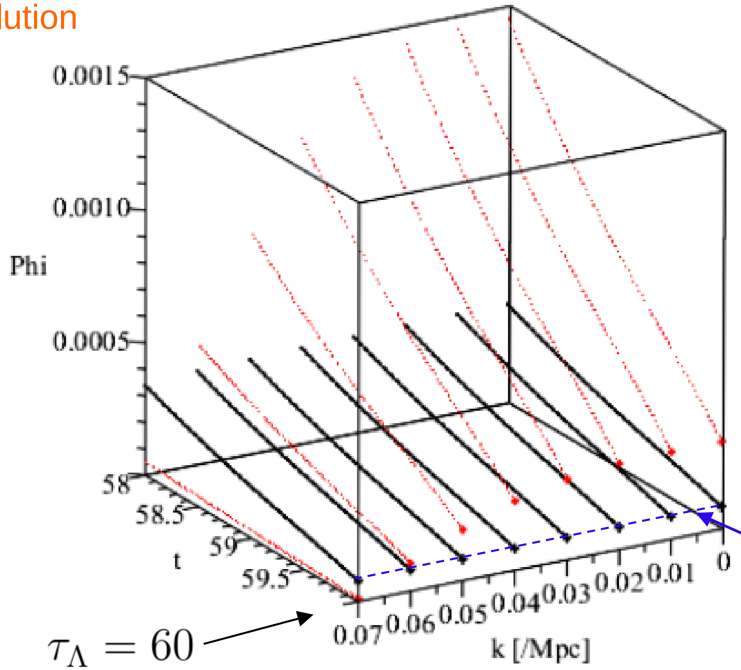
Time Evolution of Φ

Black: nonlinear evolution
Red: linear evolution



$\tau_\Lambda = 60$

Last stage of evolution



$\tau_\Lambda = 60$

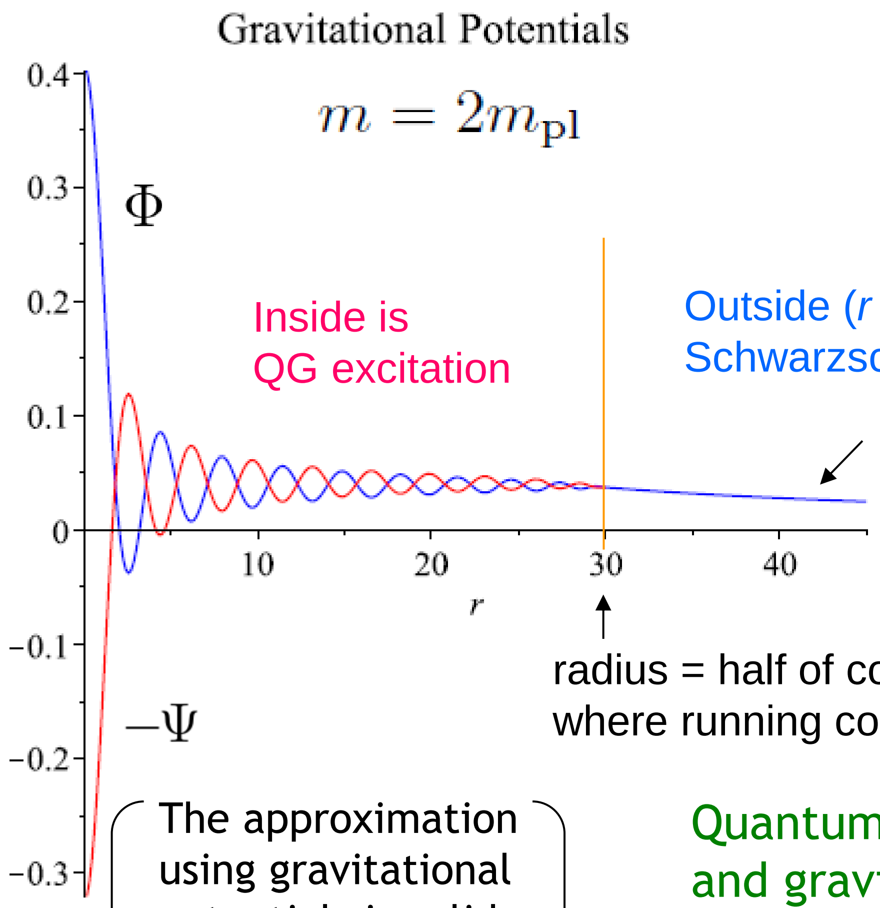
primodial spectrum

Comoving scales: $m = 0.02\,\mathrm{Mpc}^{-1}$

Final overall amplitude is adjusted by dynamical parameters β_0 and γ_1, but spectral pattern is indep. of them

Spherical and Static Excitation

[K. H., Phys. Rev. D 102 (2020) 026024]



A solution of $T_{\mu\nu} = 0$ with $T^{\rm M}_{\mu\nu} = 0$ solved under approximation that VEV of running coupling constant is replaced with a position-dependent mean field

Outside (r > 30) is Schwarzschild solution

$$\Phi = -\Psi = \frac{r_g}{2r} \qquad r_g = 2Gm\,(= 4/m_{\rm pl})$$

$$m = \int_{|x|\le\frac{\xi_\Lambda}{2}} d^3\mathbf{x}\, T^{(4)}_{00}(\mathbf{x})$$

radius = half of correlation length where running coupling diverges

$$\xi_\Lambda\,(= 1/\Lambda_{\rm QG}) \simeq 100 \times l_{\rm pl}$$

The approximation using gravitational potentials is valid only for $r_g/2\xi_\Lambda \ll 1$

Quantum gravity is activated inside excitation and gravitational fields oscillate greatly

YM Effective Action

[K. H., Phys. Rev. D 102 (2020) 125005]

Logarithmic loop corrections related to beta function are summarized into running coupling constant at all orders like

$$\Gamma_{\rm YM} = -\frac{1}{4}\int \frac{d^4q}{(2\pi)^4}\Bigg\{\frac{1}{\mathfrak{g}^2} + \left(\beta_0 + \beta_1\mathfrak{g}^2 + \beta_2\mathfrak{g}^4 + \beta_3\mathfrak{g}^6\right)\log\left(\frac{Q^2}{\mu^2}\right) - \frac{1}{2}\Big[\beta_1\beta_0\mathfrak{g}^4 + \left(2\beta_2\beta_0 + \beta_1^2\right)\mathfrak{g}^6\Big]\log^2\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{3}\beta_1\beta_0^2\mathfrak{g}^6\log^3\left(\frac{Q^2}{\mu^2}\right) + \cdots\Bigg\}F^a_{\mu\nu}F^a_{\lambda\sigma}(q)\,\eta^{\mu\lambda}\eta^{\nu\sigma}$$

$$= -\frac{1}{4}\int \frac{d^4q}{(2\pi)^4}\frac{1}{\bar{\mathfrak{g}}^2(Q)}F^a_{\mu\nu}F^a_{\lambda\sigma}(q)\,\eta^{\mu\lambda}\eta^{\nu\sigma}$$

This reflects the fact that effective action is RG inv.

where $$\bar{\mathfrak{g}}^2(Q) = \frac{1}{\beta_0\log(Q^2/\Lambda^2_{\rm QCD})}\left[1 - \frac{\beta_1\log\left[\log(Q^2/\Lambda^2_{\rm QCD})\right]}{\beta_0^2\log(Q^2/\Lambda^2_{\rm QCD})} + \frac{\beta_1^2}{\beta_0^4\log^2(Q^2/\Lambda^2_{\rm QCD})}\left\{\left(\log\left[\log\left(\frac{Q^2}{\Lambda^2_{\rm QCD}}\right)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right\}\right]$$

Note that although this is a perturbative expression satisfied for $Q > \Lambda$, this expression infers that when physical momentum Q becomes less than Λ and running coupling diverges, the gauge field action vanishes, which means that the YM dynamics disappear and confinement will occur